Question 1

Part a

Many people gave counterexamples similar to Figure 1.

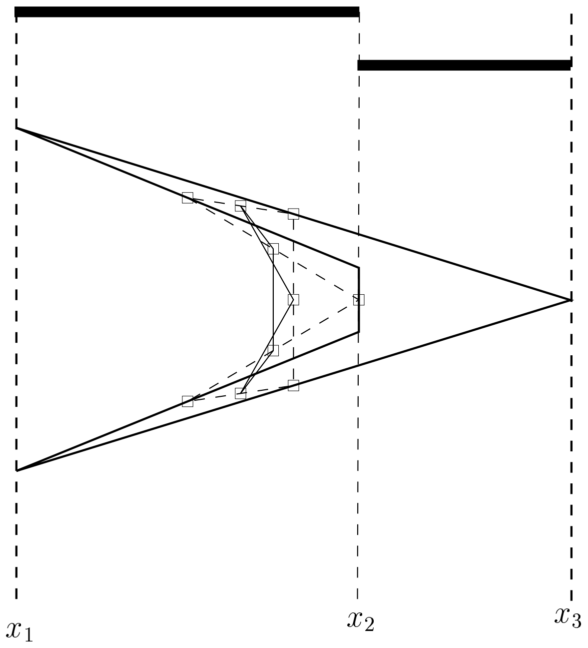

Figure 1: Illustration for Question 1

Part b

With the right proportions, the example in Figure 1 never becomes convex. We are primarily interested in x-coordinates: we argue that the ratio

$$\frac{x_2 - x_1}{x_3 - x_2} \tag{1}$$

remains constant, given a very special starting value.

Suppose

$$\frac{x_2 - x_1}{x_3 - x_2} = \frac{1+\sqrt{5}}{2} \equiv \phi \tag{2}$$

After 1 iteration, the x coordinates again fall on three lines, with x coordinates (in increasing order),

$$z_1 = (x_1 + x_2)/2$$
$$z_2 = (x_1 + x_3)/2$$
$$z_3 = x_2$$

Since $\phi > 1$, $z_2 < z_3$ and $z_1 < z_2$.

$$\begin{aligned}
\frac{z_2 - z_1}{z_3 - z_2} &= \frac{x_3 - x_2}{2x_2 - x_1 - x_3} \\
&= \frac{x_3 - x_2}{(x_2 - x_1) - (x_3 - x_2)} \\
&= \frac{1}{\phi - 1} \\
&= \frac{2}{\sqrt{5} - 1} \\
&= \frac{2(\sqrt{5}+1)}{(\sqrt{5}-1)(\sqrt{5}+1)} \\
&= \phi
\end{aligned}$$

In fact here we could have started out without knowing the proportion, and used the equation

$$\phi = \frac{1}{\phi - 1}$$

as our definition: ϕ and $-\phi$ are the only solutions to this equation.

Notice that the two vertices on the vertical line farthest to the left cannot be be connected by an edge (since both of their neighbours are strictly to the right). It follows that this figure can never become convex.

Question 2

The main observation here is that choosing the point farthest from a segment to split the chain is not always optimal. Figure 2 exhibits one of the many cases where this is true.

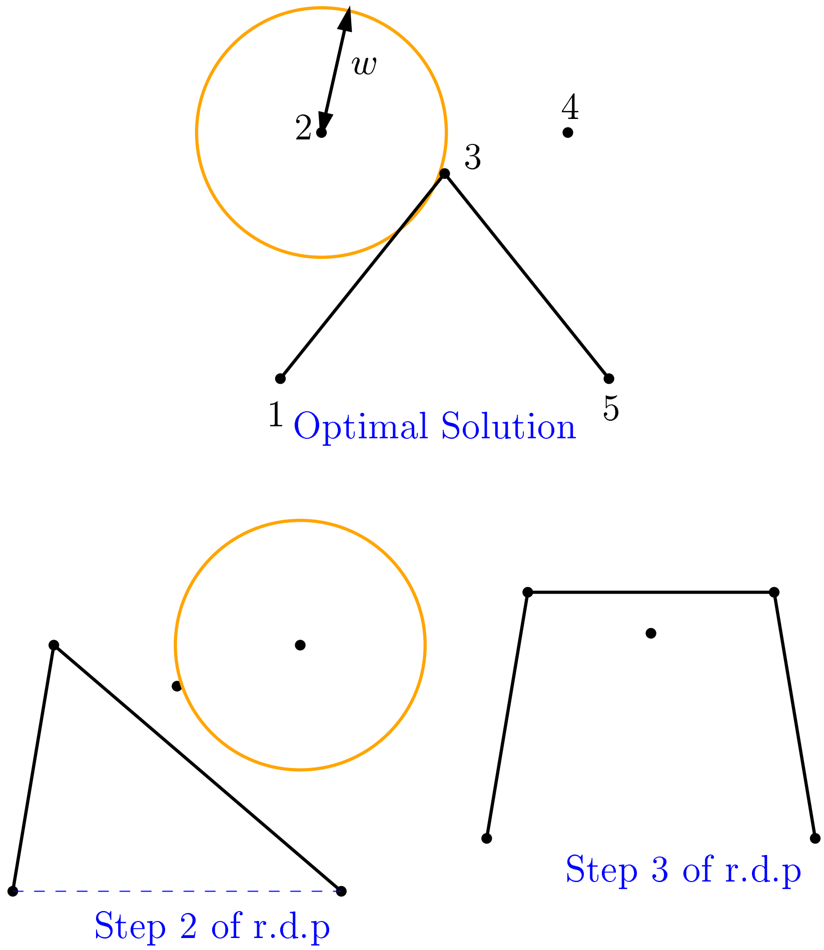

Figure 2: Illustration for Question 2

Question 3

Generally people understood that the velocity is preportional to the angle for the bisector of two line segments, and that the velocity along a parabolic bisector (for a reflex vertex and a line segment) is infinite at the closest point (at least for the case illustrated here) and decreases along the parabola. Just to make things more fun, I use a heptagon instead of a hexagon. The medial axis is (approximately) given in Figure 3. Note that the parabolic section is exagerated. In Figures 4 and 5 I have sketched successive simplifications as slower segments are deleted. One thing a few people had trouble with is the reconstruction: for every point on the (remains of) the medial axis there is a smallest circle that touches the polygon. The reconstruction is just the union of these circles.

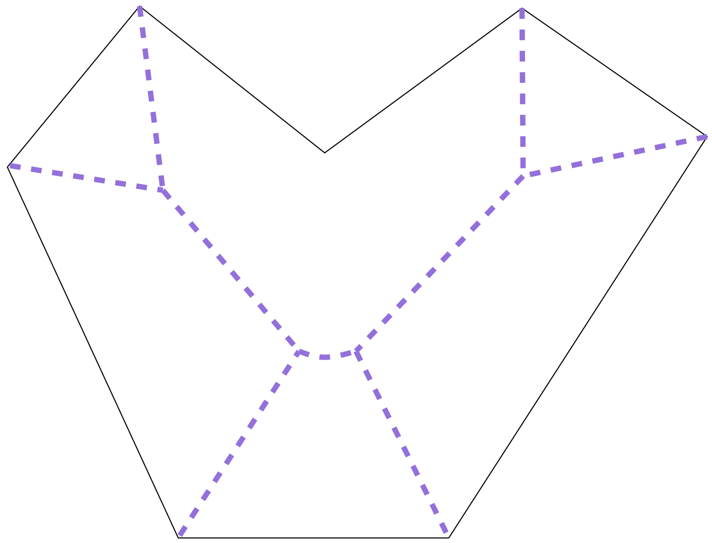

Figure 3: Medial Axis

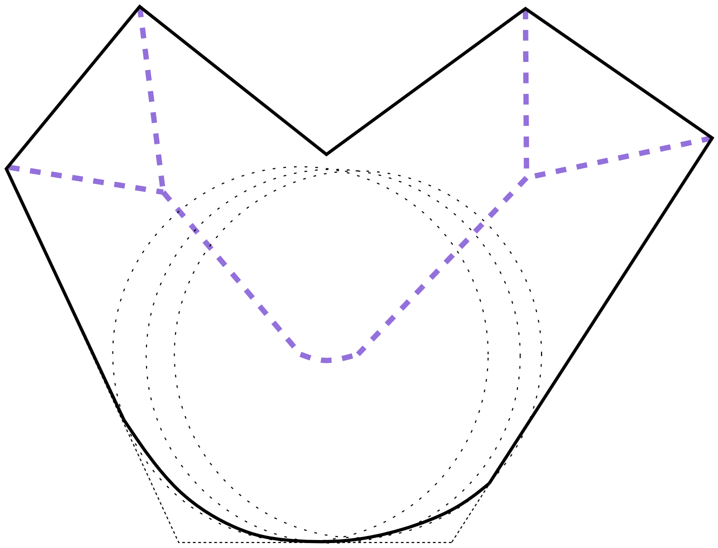

Figure 4: first simplification

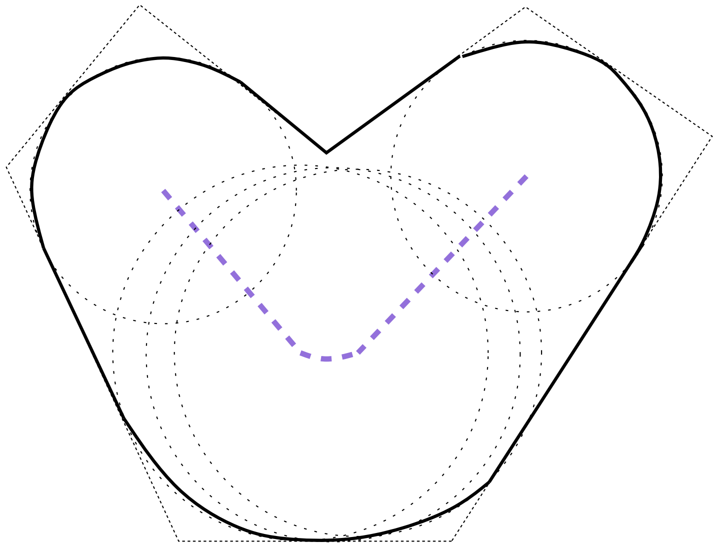

Figure 5: Second simplification