

- A.** Use the method of proof by contradiction to prove the following statements. (In each case you should also think about how a direct or contrapositive proof would work. You will find in most cases that proof by contradiction is easier.)
1. Suppose $n \in \mathbb{Z}$. If n is odd, then n^2 is odd.
 2. Suppose $n \in \mathbb{Z}$. If n^2 is odd, then n is odd.
 3. Prove that $\sqrt[3]{2}$ is irrational.
 4. Prove that $\sqrt{6}$ is irrational.
 5. Prove that $\sqrt{3}$ is irrational.
 6. If $a, b \in \mathbb{Z}$, then $a^2 - 4b - 2 \neq 0$.
 7. If $a, b \in \mathbb{Z}$, then $a^2 - 4b - 3 \neq 0$.
 8. Suppose $a, b, c \in \mathbb{Z}$. If $a^2 + b^2 = c^2$, then a or b is even.
 9. Suppose $a, b \in \mathbb{R}$. If a is rational and ab is irrational, then b is irrational.
 10. There exist no integers a and b for which $21a + 30b = 1$.
 11. There exist no integers a and b for which $18a + 6b = 1$.
 12. For every positive rational number x , there is a positive rational number y for which $y < x$.