Data Structures and Algorithms COMP-251 A

Problem Assignment #5

1. The Closest-Pair Problem

(a) Prove that a rectangle of width d and height $2d$ can contain at most six points such that any two points are at distance at least d apart.

(b) Modify the closest-pair algorithm so that it measures distance using the Manhattan metric rather than the Euclidean distance. Try to make the complexity as small as possible. Provide a complexity analysis and a proof of correctness of your algorithm.

2. Graph Embeddability

(a) Prove that a graph G is embeddable in the *plane* if and only if it is embeddable on the *sphere*.

(b) Prove that a planar embedding of a graph can be transformed into a different planar embedding such that any specified face becomes the exterior face.

3. Vertex Degree in Planar Graphs

Prove or disprove that for every positive integer n, every planar graph of n vertices contains a vertex of degree less than or equal to four. Note that this is not a multi-graph and therefore multiple edges are not allowed. For a multi-graph this problem is trivial.