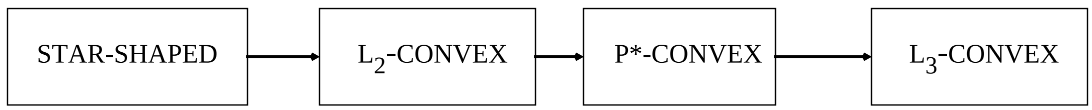

Fig. 3.3

gons considered in this note.

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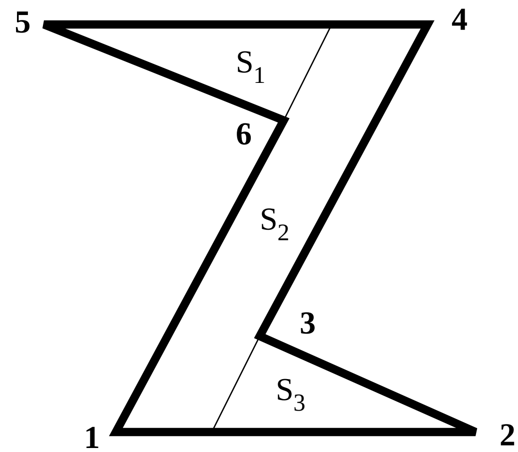

Fig. 3.1

anteed by the definition of S*(p). Finally let k be a point in the kernel of S*(p). Then clearly it follows that the path p,k,q',q lies in P and is of link-distance three. Since the choice of p and q was arbitrary it follows that P is L_3-convex. On the other hand, an L_3-convex polygon is not necessarily P*-convex, as illustrated in Fig. 3.2. Consider the point p. There is no star-shaped region S*(p) that P is weakly visible from. For S* to contain p the kernel of S*(p) must lie in triangle psq. If this kernel lies below [ss'] then q' is not visible from S*(p). On the other hand if the kernel lies above [ss'] and close enough to r so that q' is visible from S*(p) then r' becomes invisible from S*(p). Therefore we have established the following result.

Theorem 3.1: P*-convex polygons subsume L_2-convex polygons and are a subclass of L_3-convex polygons.

Fig. 3.3 illustrates the various relationships that exist between the different classes of poly-

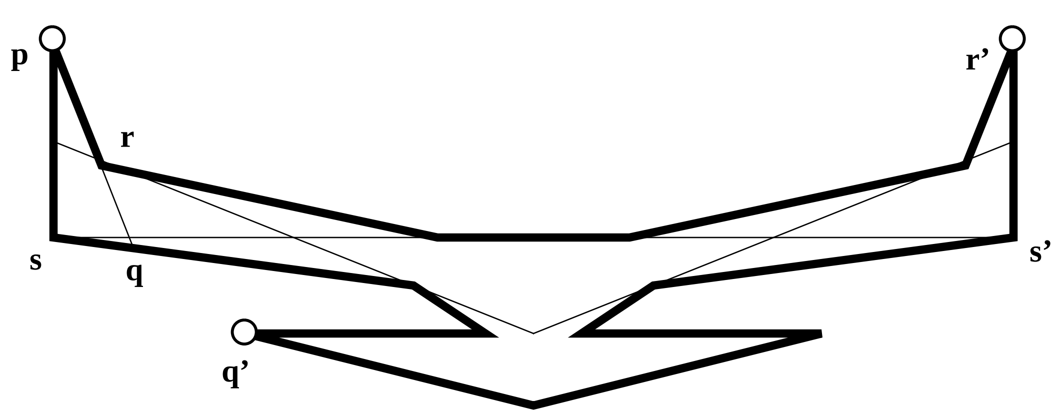

Fig. 3.2

converse also holds true this is in fact a characterization of L-convex polygons. An interesting question arises when we relax the chord L(x) traversing x to allow more general regions such as *star-shaped* regions.

2. A new characterization of L-convex polygons

Horn and Valentine [HV] characterized L-convex polygons in terms of a covering of P as expressed by the following theorem.

Theorem 2.1: (Horn & Valentine) A simple polygon P is L-convex if, and only if, P can be expressed as the sum of convex subsets of P every two of which have a point in common.

Here we provide an alternate characterization in terms of weak visibility. In the sequel let S*(x) denote the star-shaped subset of P containing x from which P is weakly visible.

Theorem 2.2: A simple polygon P is L-convex if, and only if, P has the property that for every point x in P there exists a subset S* of P such that: (1) x is contained in S*, (2) S* is star-shaped from x, and (3) P is *weakly-visible* from S*.

Proof: [only if] If P is L-convex it has the property that for every point x in P there exists a traversing chord L(x) from which P is weakly visible [HV]. Clearly L(x) satisfies the three conditions of the theorem. [if] Let x and y be any two points in P. From the weak visibility of P from S*(x) it follows that there must exist a point z in S*(x) visible from y. From the star-shapedness of S*(x) from x it follows that x and z are visible. Therefore x and y have link-distance two. Since x and y were chosen arbitrarily we have that P is L-convex. Q.E.D.

3. A new class of polygons

It is interesting to consider a further generalization by removing from condition (2) the requirement that S* be star-shaped from x. We then obtain a new class of polygons.

Definition: A simple polygon P is said to be *P*- convex* provided that every point x in P is contained in a star-shaped subset of P from which P is weakly visible.

An L-convex polygon is clearly P*-convex. However, the converse is no longer true as illustrated in Fig. 3.1. The polygon in Fig. 3.1 is not L-convex because the link-distance between vertices **2** and **5** is three. On the other hand the polygon is P*-convex. To see this let S_{12} denote the union of S_1 and S_2 and let S_{23} denote the union of S_2 and S_3. Every point x in P must lie in either region S_{12} or S_{23}, both regions are star-shaped from vertices **4** and **1**, respectively, and P is weakly visible from each such region.

We can also show that if a polygon is P*-convex it must be L_3-convex. To see this choose any two points p,q in a polygon that is P*-convex and let S*(p) be the star-shaped region in P that contains p as guaranteed by the definition. Let q' be a point in S*(p) that is visible from q as guar-

A New Characterization of L-Convex Polygons

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ABSTRACT

In 1949 Horn and Valentine [HV] showed that if each pair of points a,b in a simple polygon P could be connected by a polygonal path of length two lying in P (such polygons are termed *L-convex* polygons) then through each point x in P there exists a line segment L(x) lying in P such that for every point y in P there exists a point z in L(x) with the property that the segment yz lies in P. Since the converse also holds true this is in fact a characterization of *L-convex* polygons. We show that by relaxing L(x) from a *line-segment* to a *star-shaped* subset S(x) of P containing x we obtain a new characterization of *L-convex* polygons if S(x) is constrained to be star-shaped from x, and a new class of polygons if it is not.

1. Introduction

This note is concerned with certain link-distance properties of a simple planar polygon P having n sides. The notion of a *link-distance* between two points a, b inside P was introduced as early as 1949 by Horn and Valentine [HV]. Since then mathematicians have investigated the properties of this distance measure further in [BB] and [Va] whereas computer scientists have investigated the computational aspects [LPSSSTWY] and [Su]. The link-distance is defined as the smallest number of links (i.e., straight line segments) in a polygonal path connecting a and b within P, and turns out to be a useful metric for path planning within P when straight motion is easy to accomplish but turns are expensive. Alternately, it is the ideal metric for modeling robots that use telescopic-joint manipulators to pick and place objects in a work-space represented by a simple polygon.

A *chord* of a polygon P is a line segment [ab] contained in P such that both of its endpoints a and b are in *bd*(P). A polygon P is said to be L_2-convex (or simply L-convex) if every pair of points a,b in P have a *link-distance* of two between each other. More generally we say that P is L_k-convex if every pair of points a,b in P have link-distance k between them. L_2-convex polygons have received some attention in the computational geometry literature. In particular, Elgindy, Avis and Toussaint [EAT] have shown that if a polygon is known to be L_2-convex it can be triangulated in linear time. No such efficiency is known for arbitrary simple polygons. They also show that testing a simple polygon for L_2-convexity can be done in $O(n^2)$ time. P is said to be *weakly-visible* [AT] from a subset S of P if for every point x in P there exists a point y in S such that the line segment [xy] lies in P. Horn and Valentine [HV] have shown that if P is L-convex then for every point x in P there exists a chord that traverses x, say L(x), such that P is weakly visible from L(x). Since the